Photometric Stereo with Small Angular Variations

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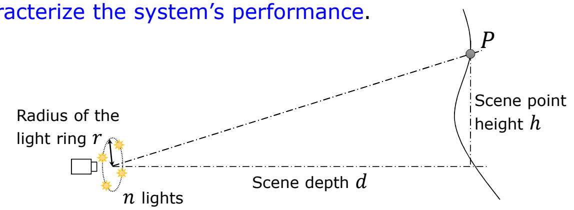
Proposed Device

We propose a compact photometric stereo system with a light ring of radius r and a camera placed at the center of the ring seeking to sense a scene whose depth d is significantly larger than r, and characterize the system's performance.



Contributions

We provide a specific dependence of the error on system parameters like

- radius of the light ring r (error $\propto 1/r^2$)
- the number of lights n (error $\propto 1/n$)
- variance of measurement noise σ^2 (error $\propto \sigma^2$)
- the mismatch between real depth d and light calibrated depth $\hat{d}$

This allows us to study the impact of various design factors on the system's performance

- tradeoff between camera quality/price (σ), compactness (r), acquisition time (n), and power (n)
- prediction of the range of the depth where error is tolerable
- confidence map of the system's performance

Sensing model

A Lambertian scene point P at the location $\mathbf{x} \in \mathbb{R}^3$ with a surface normal $\mathbf{n} \in \mathbb{R}^3$ and diffuse albedo ρ shows an intensity i under a point light source at the location $\mathbf{s} \in \mathbb{R}^3$ as:

$$i = \frac{\mathbf{l}^\top}{\|\mathbf{l}\|^3}\rho\mathbf{n}, \text{ where } \mathbf{l} = \mathbf{s} - \mathbf{x}$$

Given n (≥ 3) different lightings, we have

$$\underbrace{[i_1 \;\; i_2 \;\; \cdots \;\; i_n]^\top}_{\mathbf{i}} = \underbrace{\left[\frac{\mathbf{l}_1}{\|\mathbf{l}_1\|^3} \;\; \frac{\mathbf{l}_2}{\|\mathbf{l}_2\|^3} \;\; \cdots \;\; \frac{\mathbf{l}_n}{\|\mathbf{l}_n\|^3}\right]^\top}_{\mathbf{L}^\top} \underbrace{(\rho\mathbf{n})}_{\mathbf{b}}$$

$\mathbf{i} \in \mathbb{R}^n$ - intensity measurements, obtained from camera captured photos (known)
$\mathbf{L} \in \mathbb{R}^{3\times n}$ - light matrix including light directions and light intensities which can be calibrated (known)
$\mathbf{b} \in \mathbb{R}^3$ - albedo-scaled surface normal (unknown)

$\Rightarrow$ $\mathbf{b}$ can be computed as $\mathbf{b} = (\mathbf{L}\mathbf{L}^\top)^{-1}\mathbf{L}\mathbf{i}$

Analysis of the Device

In reality $\hat{\mathbf{b}} = (\hat{\mathbf{L}}\hat{\mathbf{L}}^\top)^{-1}\hat{\mathbf{L}}\hat{\mathbf{i}}$

$\hat{\mathbf{i}} = \mathbf{i} + \Delta\mathbf{i}_g$ (due to measurement noise)
$\hat{\mathbf{L}}$ deviates from $\mathbf{L}$ (due to depth mismatch)
then $\hat{\mathbf{b}}$ differs from the ground truth $\mathbf{b}$

In the presence of measurement noise

Estimation of $\mathbf{b}$: $\hat{\mathbf{b}} = (\mathbf{L}\mathbf{L}^\top)^{-1}\mathbf{L}\hat{\mathbf{i}} = (\mathbf{L}\mathbf{L}^\top)^{-1}\mathbf{L}(\mathbf{i} + \Delta\mathbf{i}_g)$
$= \mathbf{b} + (\mathbf{L}\mathbf{L}^\top)^{-1}\mathbf{L}\Delta\mathbf{i}_g$

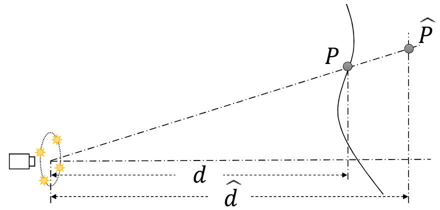
$\Delta\mathbf{i}_g \in \mathbb{R}^n$ - noise term, random variable (noise is *i.i.d.* with mean 0 and variance σ^2)

Error evaluation: $e_{l_2} = \|\hat{\mathbf{b}} - \mathbf{b}\|^2$

Error expectation: $\mathbb{E}_{\Delta\mathbf{i}_g}[e_{l_2}] = \boldsymbol{\sigma}^2(d^2+h^2)^3\dfrac{2(2d^2+h^2)}{\boldsymbol{n}\boldsymbol{r}^2 d^2}$

In the presence of calibration error

Camera assumes scene point at depth d is at light calibrated depth $\hat{d}$.

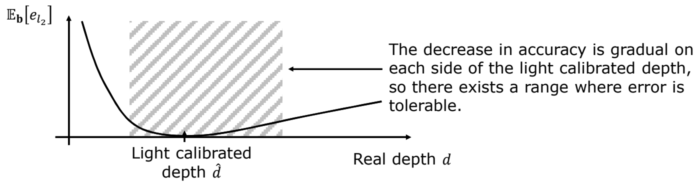


Estimation of $\mathbf{b}$: $\hat{\mathbf{b}} = (\hat{\mathbf{L}}\hat{\mathbf{L}}^\top)^{-1}\hat{\mathbf{L}}\mathbf{i} = (\hat{\mathbf{L}}\hat{\mathbf{L}}^\top)^{-1}\hat{\mathbf{L}}\mathbf{L}^\top\mathbf{b}$

$\mathbf{L} \in \mathbb{R}^{3\times n}$ - light matrix of point P
$\hat{\mathbf{L}} \in \mathbb{R}^{3\times n}$ - light matrix of point $\hat{P}$
$\mathbf{b} \in \mathbb{R}^3$ - random variable, uniformly distributed in all directions

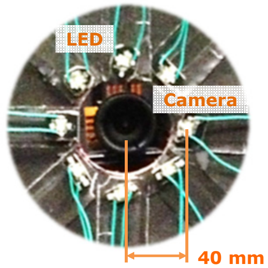
Error expectation:

$$\mathbb{E}_{\mathbf{b}}[e_{l_2}] = \frac{1}{3}\rho^2\left(\frac{\hat{d}}{d}-1\right)^2\left(2\left(\frac{\hat{d}^2}{d^2}+\frac{\hat{d}}{d}+1\right)^2+\left(\frac{\hat{d}}{d}+1\right)^2\right)$$

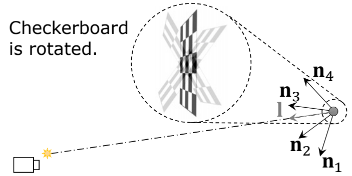


When we have both errors, the expected error is simply linear combination of the two.

Experiments



Hardware prototype

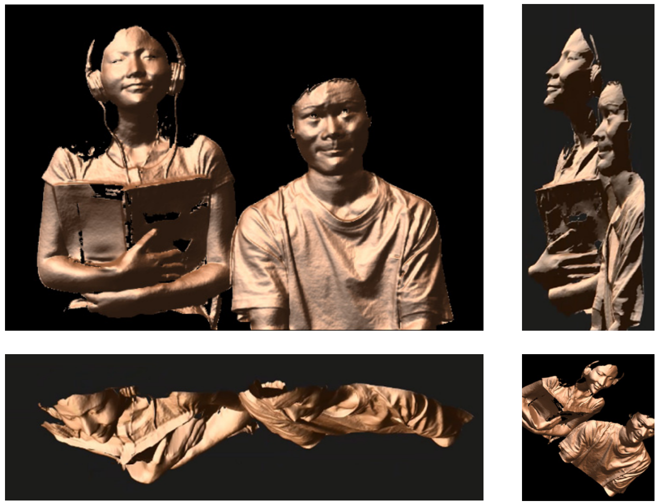


Light calibration method

We obtain the light vector from a light to a location by varying the orientation of a planar checkerboard and imaging it under the light's illumination.


(a) One of the input


(b) Surface normal map


(c) Integrated surface

Results of human scene (1m X 0.7m)

Reference

Ondrej Drbohlav and Mike Chantler, "On optimal light configurations in photometric stereo." ICCV 2005