Radon Transform Imaging: Low-Cost Video Compressive Imaging at Extreme Resolutions

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ABSTRACT

Most compressive imaging architectures rely on programmable light-modulators to obtain coded linear measurements of a signal. As a consequence, the properties of the light modulator place fundamental limits on the cost, performance, practicality, and capabilities of the compressive camera. For example, the spatial resolution of the single pixel camera is limited to that of its light modulator, which is seldom greater than 4 megapixels. In this paper, we describe a novel approach to compressive imaging that avoids the use of spatial light modulator. In its place, we use novel cylindrical optics and a rotation gantry to directly sample the Radon transform of the image focused on the sensor plane. We show that the reconstruction problem is identical to sparse tomographic recovery and we can leverage the vast literature in compressive magnetic resonance imaging (MRI) to good effect.

The proposed design has many important advantages over existing compressive cameras. First, we can achieve a resolution of $N \times N$ pixels using a sensor with N photodetectors; hence, with commercially available SWIR line-detectors with 10k pixels, we can potentially achieve spatial resolutions of 100 megapixels, a capability that is unprecedented. Second, our design is scalable more gracefully across wavebands of light since we only require sensors and optics that are optimized for the wavelengths of interest; in contrast, spatial light modulators like DMDs require expensive coatings to be effective in non-visible wavebands. Third, we can exploit properties of line-detectors including electronic shutters and pixels with large aspect ratios to optimize light throughput. On the flip side, a drawback of our approach is the need for moving components in the imaging architecture.

Keywords: Compressive Sensing, Radon Transform, Spatial Multiplexing

1. INTRODUCTION

Advances in compressive sensing (CS) has led to an impressive array of practical imagers[1–6] that enable sub-Nyquist imaging capabilities. In this paper, we focus on spatial multiplexing cameras where a low-resolution sensor is optically super-resolved to image a scene at high spatial resolution. The classical example of spatial multiplexing is the single pixel camera (SPC)[1] where a single photodetector is super-resolved using a spatial light modulator; in spite of imaging using a photodetector — which has no spatial resolving capabilities — the SPC images at the resolution of its spatial light modulator. This capability is especially powerful when high-resolution sensor arrays are expensive which is often the case in many non-visible wavebands including short- and mid-wave infrared as well as THz. In these cases, CS using the SPC and many of its multi-pixel variants[7–9] provide unique operating points where we can obtain high-resolution images and videos from inexpensive low-resolution sensors.

Most compressive imaging architectures rely on programmable spatial light modulators to obtain coded linear measurements of a signal. As a consequence, the properties of the light modulator place fundamental limits on the performance and capabilities of the compressive camera. For example, the spatial resolution of the SPC is limited to that of its light modulator, which is seldom greater than 2-4 megapixels. As a consequence, current designs for compressive imagers are restricted to imaging at relatively low-resolution images. This is a significant limitation given that it is precisely at higher resolutions that sensor arrays become prohibitively expensive.

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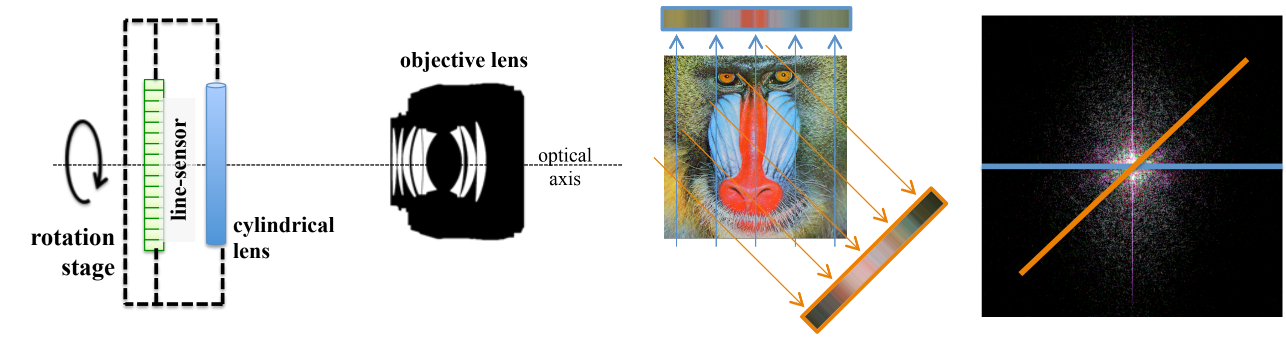


(a) Optical setup of the proposed light-modulator-free design for compressive imaging

(b) Imaging model and its representation in Fourier domain

Figure 1. Proposed imaging architecture for high-resolution compressive imaging. (a) Schematic of the proposed design which involves novel optics that capture line-integrals of the image focused by the objective lens. By rotating the sensor and cylindrical lens, we can change the orientation along which the integrals are optically computed. (b) Since each sensor measurement is a line-integral of the image focused by the objective lens, the resulting system is equivalent to radial sampling of the Fourier transform of the image via the Fourier Slice theorem. Once we obtain sufficient samples, we can obtain the image by inverting the radon transform using compressive recovery techniques.

Our approach involves a *novel compressive imaging architecture that completely avoids spatial light modulators* and their associated limitations. Instead we design an imaging architecture that optically super-resolves a line-sensor (i.e., a linear array of photodetectors) with N pixels to sense a scene at, approximately, $N \times N$ pixel resolution. This enables us to capture high-resolution images using relatively inexpensive sensors and optics. For example, using commercially-available line-sensors with 10k pixels, we can potentially sense scenes at nearly 100 megapixels — a capability that is 2-3 orders of magnitude beyond what has been achieved in prior work. This promises a novel set of approaches for high-resolution imaging in non-visible wavebands.

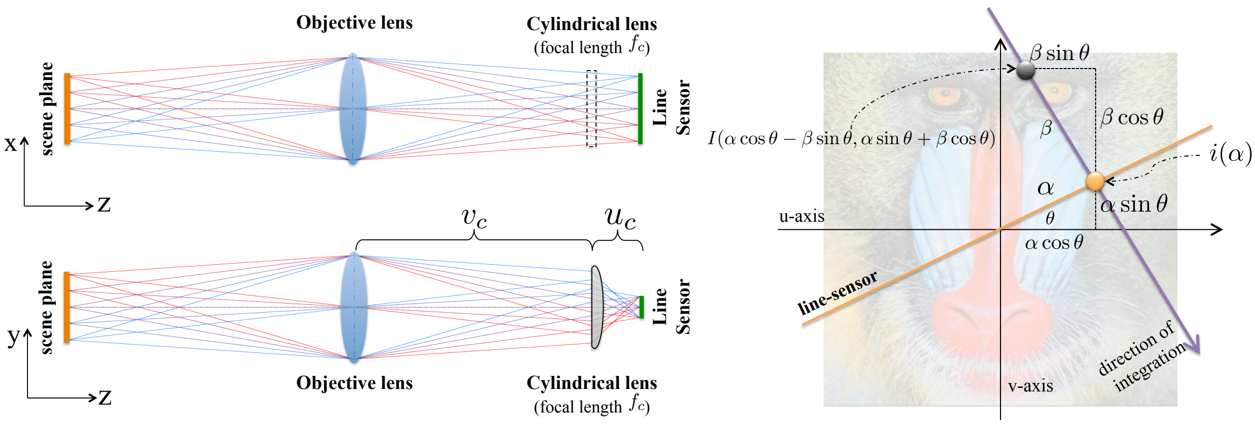
The proposed imaging architecture uses a novel optical design (see Figure 1) to directly sample the Radon transform of the image focused on the sensor plane. Specifically, each pixel on the line-sensor is designed to measure a line-integral of the image in the direction perpendicular to the axis of the sensor. Subsequently, we can measure integrals along lines of different orientation by rotating the sensor using a rotation stage. By the Fourier Slice theorem,[10] the line-integrals can be mapped to samples along a line in the Fourier transform of the image, and hence, our imaging model is identical to sub-sampling the Fourier transform of the scene. Once we obtain these measurements, we can solve for the image using a wide array of inverse techniques; it is worth nothing that the reconstruction problem is identical to sparse tomographic recovery and hence, we can leverage the vast literature in compressive MRI.[11–13]

2. RADON TRANSFORM IMAGING

In this section, we describe the image formation model underlying our imaging architecture and a compressive recovery scheme to obtain images from sub-sampling of the radon transform.

Optical setup. The optical design of imaging architecture is given in Figure 2(a). The setup consists of an objective lens, a line-sensor along with a cylindrical lens and a rotation stage. The cylindrical lens is placed in between the line-sensor and the objective lens, and its axis is aligned with the axis of the line-sensor. Finally, both the line-sensor and the cylindrical lens are jointly mounted onto a rotation stage whose axis of rotation is the optical axis of the objective lens.

The placement of the optical components are done to obtain the following scene-to-sensor mapping. The line-sensor is placed on the image plane of the objective lens, so that the scene is perfectly in focus along the axis of the line-sensor. Since the cylindrical lens' axis is aligned with that of the line-sensor, it does not perturb light rays along the direction parallel to its axis. This results in the scene being in focus along the x-axis in Figure 2(a). Along the direction perpendicular to the axis of the cylindrical lens (y-axis in Figure 2), the position and



(a) Ray diagram of the optical setup (b) Image formation

Figure 2. Design of the imaging architecture and the ensuing imaging model. (a) Ray diagrams along two orthogonal axes. The line-sensor is placed at the image plane of the objective lens. A cylindrical lens is placed in front of the line-sensor such that its axis is aligned with that of the line-sensor. The cylindrical lens does not perturb light rays along the x-axis (top-row); this results in the scene being in focus along the x-axis. Along the y-axis (bottom-row), the cylindrical lens brings the aperture plane into focus at the image plane. Hence, the scene is completely defocused along the y-axis, i.e., each line-sensor pixel integrates light along the y-axis. Finally, the cylindrical lens and line-sensor can jointly be rotated about the optical axis of the objective lens, which has the effect of changing the orientation along with light is integrated. (b) This corresponds to each line-sensor pixel integrating light along a line on the image plane. The orientation of the line-sensor, θ, controls the direction of integration and is changed via the rotation stage.

focal length of the cylindrical lens are chosen to ensure that the aperture plane of the objective lens is focused at the image plane. Hence, the scene is completely defocused along the y-axis, i.e., each line-sensor pixel integrates light along the y-axis. This is illustrated in Figure 2(a). Further, for maximum efficiency in gathering light, it is desirable that the aperture of the objective lens is magnified/shrunk to the height of the line-sensor. In this context, it is worth noting that line-sensors often have highly skewed pixel size (i.e, pixel-heights are significantly larger than pixel-widths) which allows for large light collection with no commensurate loss in resolution.

The rotation stage allows us to jointly rotate the cylindrical lens and the line-sensor and hence, it has the effect of rotating the definition of x- and y-axes in Figure 2(a). This has the effect of allowing us to choose different orientations on the image plane to integrate light along.

Imaging model. The imaging model of the optical setup can be mathematically described as follows. Let $I(u, v)$ be the image focused on the image plane of the objective lens. Suppose that the axis of the line-sensor makes an angle θ to the u-axis of the image plane (see Figure 2(b)). Then, each pixel integrates light along a line on the image plane that is perpendicular to the axis of the line-sensor.* Mathematically, the measurement made at a pixel α on the line-sensor is given as:

$$y_\theta(\alpha) = \int_{\beta=-\infty}^{+\infty} I(\alpha\cos\theta - \beta\sin\theta, \alpha\sin\theta + \beta\cos\theta) d\beta \tag{1}$$

By rotating the sensor and the cylindrical lens, we can obtain measurements corresponding to different values of θ, assuming a static or slow-moving scene. Our goal is to recover the image I from a collection of measurements $\{y_\theta, \theta \in \Theta\}$ where $\Theta = \{\theta_1, \theta_2, \ldots, \theta_M\}$ is the collection of orientations at which measurements are made.

Connections to Radon transform. The collection $\{y_\theta, \theta \in (0, \pi]\}$ is referred to as the Radon transform[14,15] of the image $I(u, v)$. The Radon transform arises commonly in the context of medical imaging where it is often referred to as tomography. Further, via the Fourier Slice theorem, it is easily shown that the Fourier transform

*For simplicity, we are assuming a sensor with no pixellation and with no field of view constraints.

of y_θ is simply a sampling of the 2D Fourier transform of I along a pre-specified orientation that is determined by θ (see Figure 1(b)). This allows for fast and numerically stable implementations of the Radon transform.

Recovery algorithms. There are multiple techniques that robustly determine the image I from a sampling, of the Radon transform, $\{y_\theta, \theta \in \Theta\}$. When the set Θ is dense, so that the inverse problem is over-determined, we could use filtered backprojection techniques to recover the image without significant artifacts. Invariably, in this paper, we operate in the compressive regime where the set Θ is sparsely selected; in this context, it is common to regularize the inverse problem using image priors. To this end, we use the following framework to recover the image.

$$\min_I \|\mathbf{s}\|_1, \quad \text{s.t.} \quad \mathbf{s} = \Psi(I), \quad \forall \theta \in \Theta, \|y_\theta - A_\theta(I)\|_2 \leq \epsilon \tag{2}$$

Here, $\Psi(I)$ refers to the wavelet decomposition of the image I, and $A_\theta(I)$ simulates the measurement process when the line-sensor is oriented along θ. The objective in (2) minimizes the ℓ_1-norm of $\mathbf{s}$ and hence, promotes sparse wavelet coefficients in the solution. Finally, the parameter $\epsilon > 0$ captures the measurement noise level.

The optimization problem in (2) is referred to as basis pursuit denoising[16] and there are many efficient optimization packages for solving it. For the results in this paper, we use SPG-L1.[17] We also note that it is possible to implement the operator A_θ in numerically robust and computationally efficient manner using the Fourier slice theorem.

Simulations. We provide brief empirical validation of the proposed concept using simulations. In Figure 3, we show reconstruction performance for varying compression levels, defined as the ratio of the dimensionality of the image to the number of measurements. Given the ground truth image I and the reconstructed image $\widehat{I}$, we measure reconstruction performance in terms of SNR (in dB) defined as

$$\text{SNR}(\widehat{I}) = -20\log_{10}\left(\frac{\|I - \widehat{I}\|}{\|I\|}\right).$$

We also characterize how the performance varies with the resolution at which the scene is imaged and observe that, for the compression level, the performance improves when we sense a scene at higher resolution. This indicates that there are significant improvements to be derived when we image a scene at very high-resolution — an observation that is consistent with prior work.[18]

Advantages of Radon Transform Imaging. The proposed design has many important advantages over current designs for compressive cameras.

- *Resolution.* We can achieve a resolution of $N \times N$ pixels using a sensor with N photodetectors; hence, with commercially available line-detectors with 10,000 pixels, we can achieve spatial resolutions of nearly 100 megapixels — this capability is unprecedented even in the visible spectrum.

- *Cost.* The proposed design does not use a spatial light modulator and hence, can be inexpensive especially since a rotation gantry can be made at very low cost. Further, depending on application scenario, the sensor could even be put on existing rotating structures — for example, rotor blades of an airplane. In contrast, while alternative designs like the SPC allow for the use of inexpensive sensors, they still require a high-resolution light modulator which can be just as expensive to build in certain wavelengths as a high-resolution sensor.

- *Light throughput.* Our light efficiency can be as large as 100% with carefully designed cylindrical optics since commercially-available line-sensors have tall pixels with highly-skewed aspect ratios. In contrast, DMD-based designs lose at least half the light due to the binary spatial coding.

- *Optical alignment.* In contrast to DMD-based designs that require off-axis alignment and the use of Schiemplug principle,[1,19] our design is entirely along a single optical axis. This makes alignment significantly easier.

(a) (b)

Figure 3. Performance of the proposed system using simulations. (a) We characterize reconstruction SNR as a function of the compression ratio, measured in terms of the ratio of the image dimensionality to the number of measurements, for different image resolutions. We observe that, for the same compression, reconstruction performance increases with resolution. Shown in the plot are average statistics over 10 trials. (b) We show reconstruction results at various resolutions at a compression of 12×. Overlaid on each image is the resolution of the result as well as its reconstruction SNR.

- *Computational scalability.* By the Fourier slice theorem, the 1D Fourier transform of each line-sensor measurement is the sampling along a line of the 2D Fourier transform of the image. As a consequence, the measurement operator and its transpose both enjoy very fast implementations via the FFT.
- *Scaling across spectral wavebands.* Our design scales is capable of operating in a broad range of wavelengths since we only require sensors and optics that are optimized for the wavelengths of interest; in contrast, spatial light modulators like DMDs require expensive coatings to be effective in non-visible wavebands.
- *Commercially available line-detectors* have been optimized for spectroscopy and, as a consequence, provide properties that are very favorable to CS. These include high dynamic range, high sensitivity, and pixels with very large aspect ratios that enables larger light collection capability.

Limitations. A key limitation of our approach is the need for moving components as part of the imaging architecture; however, it is worth nothing that commercially-available rotation stages can be both extremely precise and fast. Our approach also requires that the scene remain static or slow-moving over the period of time over which we acquire the compressive measurements. This limitation can potentially be addressed via the use of video compressive sensing techniques.[6,20]

3. CONCLUSION

In this paper, we introduced a novel compressive imaging architecture that forgoes the use of spatial light modulator but instead uses novel optics to directly sample the radon transform of the image of a scene. As a consequence, the proposed design is truly capable of imaging at extremely high-resolutions — a capability that is beyond the reach of current compressive imagers that rely on spatial light modulators. This, coupled with the empirical observation that performance of CS often increases when we sense scenes at very high spatial resolutions, opens up a novel design space for enabling the next-generation of compressive cameras.

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